

**Lösungsvorschlag zur Sitzung am 27.04.2020 – De Morgansche Gesetze, Halbaddierer, Flip-Flop**

Abgabe auf Ilias bis 04.05.2020, 10.00 Uhr

**Aufgabe 1**

Vereinfachen Sie die Gleichung unter Berücksichtigung der Gesetze De Morgans:

$$\neg(\neg A \vee B) = ?$$

Beweisen Sie die Gültigkeit der Umformung, indem Sie die entsprechende Wahrheitstabelle erstellen.

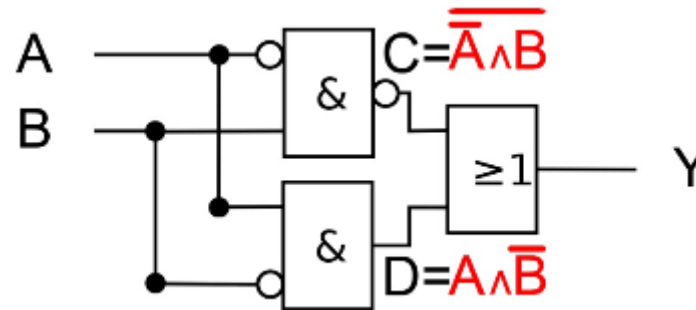
**Lösung:**

$$\neg(\neg A \vee B) = \overline{\overline{\overline{A \vee B}}} = \overline{\overline{A \vee B}} = \overline{A \wedge \overline{B}} = A \wedge \overline{B}$$

| A | B | $\neg A$ | $\neg B$ | $\neg A \vee B$ | $\neg(\neg A \vee B)$ | $A \wedge \neg B$ |
|---|---|----------|----------|-----------------|-----------------------|-------------------|
| 0 | 0 | 1        | 1        | 1               | 0                     | 0                 |
| 0 | 1 | 1        | 0        | 1               | 0                     | 0                 |
| 1 | 0 | 0        | 1        | 0               | 1                     | 1                 |
| 1 | 1 | 0        | 0        | 1               | 0                     | 0                 |

**Aufgabe 2**

Bestimmen Sie die Wahrheitstabelle für die folgende Schaltung und beschreiben Sie die Schaltung mit Hilfe einer Gleichung.

**Lösung:**

| A | B | $\neg A$ | $\neg B$ | $\neg A \wedge B$ | $C = \neg(\neg A \wedge B)$ | $D = A \wedge \neg B$ | $Y = \neg(\neg A \wedge B) \vee (A \wedge \neg B)$ |
|---|---|----------|----------|-------------------|-----------------------------|-----------------------|--|
| 0 | 0 | 1        | 1        | 0                 | 1                           | 0                     | 1  |
| 0 | 1 | 1        | 0        | 1                 | 0                           | 0                     | 0  |
| 1 | 0 | 0        | 1        | 0                 | 1                           | 1                     | 1  |
| 1 | 1 | 0        | 0        | 0                 | 1                           | 0                     | 1  |

$$Y = (\neg A \wedge \neg B) \vee (A \wedge \neg B) \vee (A \wedge B)$$

**Aufgabe 3**

Formen Sie folgende Ausdrücke um. Überprüfen Sie anschließend Ihren Ausdruck auf Äquivalenz.

Umformung UND zu ODER:

$$Z = A \wedge B \wedge C \wedge D$$

Umformung ODER zu UND:

$$Z = \bar{A} \vee \bar{B} \vee C \vee \bar{D}$$

**Lösung:**

UND zu ODER:

$$\begin{aligned} Z &= A \wedge B \wedge C \wedge D \\ &= \overline{\overline{A \wedge B \wedge C \wedge D}} \\ &= \overline{\bar{A} \vee \bar{B} \vee \bar{C} \vee \bar{D}} \end{aligned}$$

ODER zu UND:

$$\begin{aligned} Z &= \bar{A} \vee \bar{B} \vee C \vee \bar{D} \\ &= \overline{\overline{\bar{A} \vee \bar{B} \vee C \vee \bar{D}}} \\ &= \overline{A \wedge B \wedge \bar{C} \wedge D} \end{aligned}$$

**Lösung:**

a)

| A | B | C | D | $\neg A$ | $\neg B$ | $\neg C$ | $\neg D$ | $A \wedge B$ | $A \wedge B \wedge C$ | $A \wedge B \wedge C \wedge D$ | $\neg A \vee \neg B$ | $\neg A \vee \neg B \vee \neg C$ | $\neg A \vee \neg B \vee \neg C \vee \neg D$ | $\neg(\neg A \vee \neg B \vee \neg C \vee \neg D)$ |
|---|---|---|---|----------|----------|----------|----------|--------------|-----------------------|--------------------------------|----------------------|----------------------------------|--|--|
| 0 | 0 | 0 | 0 | 1        | 1        | 1        | 1        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 0 | 0 | 1 | 1        | 1        | 1        | 0        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 0 | 1 | 0 | 1        | 1        | 0        | 1        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 0 | 1 | 1 | 1        | 1        | 0        | 0        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 1 | 0 | 0 | 1        | 0        | 1        | 1        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 1 | 0 | 1 | 1        | 0        | 1        | 0        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 1 | 1 | 0 | 1        | 0        | 0        | 1        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 0 | 1 | 1 | 1 | 1        | 0        | 0        | 0        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 1 | 0 | 0 | 0 | 0        | 1        | 1        | 1        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 1 | 0 | 0 | 1 | 0        | 1        | 1        | 0        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 1 | 0 | 1 | 0 | 0        | 1        | 0        | 1        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 1 | 0 | 1 | 1 | 0        | 1        | 0        | 0        | 0            | 0                     | 0                              | 1                    | 1                                | 1  | 0  |
| 1 | 1 | 0 | 0 | 0        | 0        | 1        | 1        | 1            | 0                     | 0                              | 0                    | 1                                | 1  | 0  |
| 1 | 1 | 0 | 1 | 0        | 0        | 1        | 0        | 1            | 0                     | 0                              | 0                    | 1                                | 1  | 0  |
| 1 | 1 | 1 | 0 | 0        | 0        | 0        | 1        | 1            | 1                     | 0                              | 0                    | 0                                | 1  | 0  |
| 1 | 1 | 1 | 1 | 0        | 0        | 0        | 0        | 1            | 1                     | 1                              | 0                    | 0                                | 0  | 1  |

b)

| A | B | C | D | $\neg A$ | $\neg B$ | $\neg C$ | $\neg D$ | $(\neg A) \vee (\neg B)$ | $(\neg A) \vee (\neg B) \vee C$ | $(\neg A) \vee (\neg B) \vee C \vee (\neg D)$ | $A \wedge B$ | $A \wedge B \wedge (\neg C)$ | $A \wedge B \wedge (\neg C) \wedge D$ | $\neg(A \wedge B \wedge (\neg C) \wedge D)$ |
|---|---|---|---|----------|----------|----------|----------|--------------------------|---------------------------------|---|--------------|------------------------------|---------------------------------------|---|
| 0 | 0 | 0 | 0 | 1        | 1        | 1        | 1        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 0 | 0 | 1 | 1        | 1        | 1        | 0        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 0 | 1 | 0 | 1        | 1        | 0        | 1        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 0 | 1 | 1 | 1        | 1        | 0        | 0        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 1 | 0 | 0 | 1        | 0        | 1        | 1        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 1 | 0 | 1 | 1        | 0        | 1        | 0        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 1 | 1 | 0 | 1        | 0        | 0        | 1        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 0 | 1 | 1 | 1 | 1        | 0        | 0        | 0        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 1 | 0 | 0 | 0 | 0        | 1        | 1        | 1        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 1 | 0 | 0 | 1 | 0        | 1        | 1        | 0        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 1 | 0 | 1 | 0 | 0        | 1        | 0        | 1        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 1 | 0 | 1 | 1 | 0        | 1        | 0        | 0        | 1                        | 1                               | 1   | 0            | 0                            | 0                                     | 1   |
| 1 | 1 | 0 | 0 | 0        | 0        | 1        | 1        | 0                        | 1                               | 1   | 1            | 1                            | 0                                     | 1   |
| 1 | 1 | 0 | 1 | 0        | 0        | 1        | 0        | 0                        | 0                               | 0   | 1            | 1                            | 1                                     | 0   |
| 1 | 1 | 1 | 0 | 0        | 0        | 0        | 1        | 0                        | 1                               | 1   | 1            | 0                            | 0                                     | 1   |
| 1 | 1 | 1 | 1 | 0        | 0        | 0        | 0        | 0                        | 0                               | 0   | 1            | 0                            | 0                                     | 1   |

**Aufgabe 4**

Zeigen Sie, dass folgende nach den Morganschen Gesetzen umgeformte Ausdrücke zueinander äquivalent sind.

Verwenden Sie hierzu Wahrheitstabellen.

a)  $A \Leftrightarrow \neg(\neg A)$

b)  $\overline{A \wedge B} \Leftrightarrow \overline{A} \vee \overline{B}$

c)  $\overline{A \vee B} \Leftrightarrow \overline{A} \wedge \overline{B}$

d)  $\overline{A} \wedge \overline{B} \wedge \overline{C} \Leftrightarrow \overline{A \vee B \vee C}$

**Lösung:**

a)

| <b>A</b> | <b><math>\neg A</math></b> | <b><math>\neg(\neg A)</math></b> |
|----------|----------------------------|----------------------------------|
| 1        | 0                          | 1                                |
| 0        | 1                          | 0                                |

b)

| <b>A</b> | <b>B</b> | <b><math>A \wedge B</math></b> | <b><math>\neg(A \wedge B)</math></b> | <b><math>\neg A</math></b> | <b><math>\neg B</math></b> | <b><math>\neg A \vee \neg B</math></b> |
|----------|----------|--------------------------------|--------------------------------------|----------------------------|----------------------------|--|
| 0        | 0        | 0                              | 1                                    | 1                          | 1                          | 1                                      |
| 0        | 1        | 0                              | 1                                    | 1                          | 0                          | 1                                      |
| 1        | 0        | 0                              | 1                                    | 0                          | 1                          | 1                                      |
| 1        | 1        | 1                              | 0                                    | 0                          | 0                          | 0                                      |

c)

| <b>A</b> | <b>B</b> | <b>A ∨ B</b> | <b>¬(A ∨ B)</b> | <b>¬A</b> | <b>¬B</b> | <b>¬A ∧ ¬B</b> |
|----------|----------|--------------|-----------------|-----------|-----------|----------------|
| 0        | 0        | 0            | 1               | 1         | 1         | 1              |
| 0        | 1        | 1            | 0               | 1         | 0         | 0              |
| 1        | 0        | 1            | 0               | 0         | 1         | 0              |
| 1        | 1        | 1            | 0               | 0         | 0         | 0              |

d)

| <b>A</b> | <b>B</b> | <b>C</b> | <b>A ∨ B</b> | <b>A ∨ B ∨ C</b> | <b>¬(A ∨ B ∨ C)</b> | <b>¬A</b> | <b>¬B</b> | <b>¬C</b> | <b>¬A ∧ ¬B</b> | <b>¬A ∧ ¬B ∧ ¬C</b> |
|----------|----------|----------|--------------|------------------|---------------------|-----------|-----------|-----------|----------------|---------------------|
| 0        | 0        | 0        | 0            | 0                | 1                   | 1         | 1         | 1         | 1              | 1                   |
| 0        | 0        | 1        | 0            | 1                | 0                   | 1         | 1         | 0         | 1              | 0                   |
| 0        | 1        | 0        | 1            | 1                | 0                   | 1         | 0         | 1         | 0              | 0                   |
| 0        | 1        | 1        | 1            | 1                | 0                   | 1         | 0         | 0         | 0              | 0                   |
| 1        | 0        | 0        | 1            | 1                | 0                   | 0         | 1         | 1         | 0              | 0                   |
| 1        | 0        | 1        | 1            | 1                | 0                   | 0         | 1         | 0         | 0              | 0                   |
| 1        | 1        | 0        | 1            | 1                | 0                   | 0         | 0         | 1         | 0              | 0                   |
| 1        | 1        | 1        | 1            | 1                | 0                   | 0         | 0         | 0         | 0              | 0                   |