Recap

Second to last Tuesday

- Machine Learning Experiment
- Convert data set into correct format
- Run machine learning workflow
 - weka.classifiers.Evaluation
 - weka.classifiers.bayes.NaiveBayes
- Last Tuesday
 - Our first ML algorithm: Naive Bayes
 - Based on p(c|f) probability of class c given feature value f
 - Naive: Assumes that features are independent of each other
 - This is (usually) not the case

 $\begin{array}{l} \mbox{Decision Trees} \\ \mbox{Sprachverarbeitung (VL + <math>\ddot{U})} \end{array}$

Nils Reiter

May 25, 2023









- What are the instances?
 - Situations we are in (this is not really automatisable)



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- What are the features?



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 - Situations we are in (this is not really automatisable)
- What are the features?
 - Consciousness
 - Clothing situation
 - Promises made
 - Whether we are driving



Trees

Well-established data structure in CS



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 - some value and
 - ▶ a (possibly empty) set of children
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Prediction Model



- Each non-leaf node in the tree represents one feature
- Each leaf node represents a class label
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- Each leaf node represents a class label
- Each branch at this node represents one possible feature value
 - Number of branches = $|v(f_i)|$ (number of possible values)
- Make a prediction for x:
 - 1. Start at root node
 - 2. If it's a leaf node
 - assign the class label
 - 3. Else
 - Check node which feature is to be tested (f_i)
 - Extract $f_i(x)$
 - Follow corresponding branch
 - Go to 2

argmax p(c(F)

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- 1.~ Start with the full data set $D_{\rm train}$ as D
- 2. If D only contains members of a single class:

Done.

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 - Select a feature f_i
 - Extract feature values of all instances in D
 - Split the data set according to f_i : $D = D_a \cup D_b \cup D_c \dots$ $D_\alpha = \{x \in D | f_i(x) = \alpha\}, \quad a, b, c \in v(f_i)$
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C, QS

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Remaining question: How to select features?



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- "Homogeneity"
 - ► Increase $\{ \spadesuit \spadesuit \spadesuit \heartsuit \} = \{ \heartsuit \} \cup \{ \clubsuit \spadesuit \}$
 - No increase
 - $\{ \bigstar \spadesuit \clubsuit \heartsuit \} = \{ \clubsuit \} \cup \{ \clubsuit \clubsuit \heartsuit \}$

$|G_1 = F_0 - (F_2 + E_1)/2$

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Homogeneity: Entropy/information



Shannon (1948)

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- Homogeneity: Entropy/information

Shannon (1948)

- ▶ Rule: Always select the feature with the highest *information gain* (IG)
 - (= the highest reduction in entropy = the highest increase in homogeneity)



- Measures the amount of uncertainty
- How uncertain is the next symbol in these sequences?
 - 🕨 ааааааааааааа 🥧 🔥



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 - nmkfjigeahldcb 14 symbols, very uncertain
- Certainty depends on number of different symbols and on their distribution

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i)$$

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entropy of random variable X

$$\int_{i=1}^{n} \frac{1}{p(x_i) \log_b p(x_i)} \log_b p(x_i)$$
entropy of random variable X

$$\int_{i=1}^{n} \frac{1}{p(x_i) \log_b p(x_i)} \log_b (x_i) = y$$
exactly if

$$b^y = x:$$

$$2^5 = 32 \Leftrightarrow \log_2 32 = 5$$

Interpretation

Entropy is the average number of bits^{*} we need to specify an outcome of the random variable (* for b = 2)

Entropy (Shannon, 1948) Examples

$$\begin{split} H(\{ \clubsuit \clubsuit \clubsuit \}) &= -\frac{4}{4} \log_2 \frac{4}{4} = 0 \\ H(\{ \clubsuit \clubsuit \clubsuit \heartsuit \}) &= -\left(\underbrace{\frac{3}{4} \log_2 \frac{3}{4}}_{\bigstar} + \underbrace{\frac{1}{4} \log_2 \frac{1}{4}}_{\heartsuit}\right) = 0.811 \\ H(\{ \clubsuit \clubsuit \heartsuit \heartsuit \}) &= \ldots = 1 = H(\{ \clubsuit \clubsuit \clubsuit \heartsuit \heartsuit \heartsuit \}) = \ldots \end{split}$$

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Point-wise Mutual Information MS99, pp. 178 ff.

Point-wise: Statement about values of random variable (i.e., occurrence of specific word)

Non-pointwise mutual information makes a statement about random variables themselves

Mutual: Symmetric

One word provides information to the next and vice versa

$$\begin{split} I(w_1, w_2) &= \log_2 \frac{p(w_1, w_2)}{p(w_1) p(w_2)} \\ p(w_i) &= \text{Probability of word } w_i \\ p(w_i, w_j) &= \text{Probability of both words appearing together, up to a certain distance} \\ \log_2 x = y &\equiv 2^y = x \end{split}$$

Potilila

Entropy Mutual Information

- Entropy: Amount of uncertainty in a random variable
 - Joint entropy: Amount of uncertainty in two random variables
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 - Reduction of entropy in one random variable by knowing about the other
 - $MI(X, Y) = H(X) H(X|Y) = H(Y) H(Y|X) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}$

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•
$$I(w_1, w_2) = \log_2 \frac{p(w_1, w_2)}{p(w_1)p(w_2)}$$

Manning/Schütze, 1999, 67



Feature Selection

$$\begin{array}{rcl} H(\{ \bigstar \bigstar \bigstar \heartsuit \}) &=& H([3,1]) = 0.562 \\ H(\{ \heartsuit \}) &=& H([1]) = 0 \\ H(\{ \bigstar \bigstar \bigstar \}) &=& H([3]) = 0 \end{array}$$

$$\{ \bigstar \spadesuit \heartsuit \heartsuit \}$$

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$$H(\{ \bigstar \spadesuit \clubsuit \heartsuit \}) = H([3,1]) = 0.562$$

$$H(\{ \bigstar \rbrace) = H([1]) = 0$$

$$H(\{ \clubsuit \heartsuit \}) = H([2,1]) = 0.637$$

Feature Selection

{**\$**
{**\$**
{
$$\heartsuit$$
} {**\$**
{ \heartsuit } {**\$**
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$H(\{ \clubsuit \clubsuit \clubsuit \heartsuit\})$	=	H([3,1]) = 0.562	$H(\{ \spadesuit \spadesuit \spadesuit \heartsuit \})$	=	H([3,1]) = 0.562
$H(\{\heartsuit\})$	=	H([1]) = 0	$H(\{ \clubsuit\})$	=	H([1]) = 0
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$$IG(f_1) = H(\{ \clubsuit \clubsuit \clubsuit \heartsuit \}) - \varnothing (H(\{\heartsuit\}), H(\{ \clubsuit \clubsuit \clubsuit \}))$$

= 0.562 - 0 = 0.562
$$IG(f_2) = H(\{ \clubsuit \clubsuit \clubsuit \heartsuit \}) - \varnothing (H(\{ \clubsuit \}), H(\{ \clubsuit \clubsuit \heartsuit \}))$$

= 0.562 - $(\frac{3}{4}0.637 + \frac{1}{4}0)$
= 0.562 - 0.562 - 0.477 = 0.085

Feature Selection using Entropy

- ► We calculate entropy for the target class
- But in different sub sets of the data set

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Listing 2: Feature selection in pseudo code for a data set D

```
function select feature(D):
    base_entropy = entropy(D)
2
    ig map = \{\}
3
4
    foreach feature f:
5
      weighted_feature_entropy = 0
6
      foreach feature value v:
        D_v = subset of D with all instances that have the value v
7
8
        sub entropy = entropy(D v)
        sub_size = length(D_v)
9
        weighted_feature_entropy = weighted_feature_entropy + ( sub_entropy * sub_size )
10
      information gain = base entropy - ( (weighted feature entropy) / length(D) )
11
      ig_map.put(f, information_gain)
12
    return maximum from ig_map
13
```

J. Ross Quinlan (1986). »Induction of Decision Trees«. In: *Machine Learning* 1.1, pp. 81–106. DOI: 10.1007/BF00116251

Limitations

- Only categorical attributes
- Cannot handle missing values
- Tends to overfit: »In my experience, almost all decision trees can benefit from simplification« (Quinlan, 1993, 36)
 - Even today, overfitting is a huge challenge for ML algorithms!

 \Rightarrow Extension: C4.5

(Quinlan, 1993)

Subsection 1

Example: Spam Classification



Data set (the same as last week)

▶ Data set: 100 e-mails, manually classified as spam or not spam (50/50)

- Classes $C = \{ true/1, false/0 \}$
- Features: Presence of each of these tokens (manually selected): >casino<, >enlargement<, >meeting<, >profit<, >super<, >text<, >xxx

Mail	>casino∢	>enlargement<	>meeting <	>profit (super	>text()XXX(С
1	1	1	0	0	1	1	1	0
2	0	1	0	1	0	0	0	1
3	1	0	1	0	1	0	0	0
4	1	1	1	0	0	0	0	0
5	0	1	1	0	0	1	1	1
:	:	: :	:	:	:		:	
		•						· ·

First step: Use the full data set

H(full data set) = 1

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$$\begin{array}{rcl} H(\mbox{full data set}) &=& 1\\ H(\mbox{λcasino(=1)$} &=& 0.9991\\ H(\mbox{$\lambda$casino(=0)$} &=& 0.9985 \end{array}$$

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Next step: Use the data set *after* application of the first selected feature \Rightarrow profit (= 0

$$H(\text{data set}) = 0.99403$$

 $H(\text{bcasino}(=1)) = 0.9910$
 $H(\text{bcasino}(=0)) = 0.9963$
 $IG(\text{bcasino}()) = 0.00029$
 $IG(\text{btext}()) = 0.01151$



Next step: Use the data set *after* application of the first selected feature $\Rightarrow profit = 0$ $\Rightarrow profit = 1$

$$\begin{array}{rcl} H({\rm data\ set}) &=& 0.99403\\ H({\rm b}{\rm casino}\,(=1) &=& 0.9910\\ H({\rm b}{\rm casino}\,(=0) &=& 0.9963\\ IG({\rm b}{\rm casino}\,() &=& 0.00029\\ IG({\rm b}{\rm text}\,() &=& 0.01151 \end{array}$$

H(data set) = 0.99107

$$H(\operatorname{casino} = 1) = 0.9366$$

$$H($$
 casino $(=0) = 1$

$$IG($$
 casino() = 0.0150

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 meeting() = 0.00029



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Next step: Use the data set after application of the first two layers of selected features



Section 1

Summary

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- Naive Bayes in Weka
- Decision Tree
 - Transparent prediction model: Easy to apply by humans
 - Learning algorithm
 - Recursively split the training data set according to features
 - Use information gain to maximize the homogeneity in the sub sets
 - Compared with Naive Bayes
 - Feature dependence modeled through tree structure
 - ▶ DT in Weka: Try for yourselves! ☺