Ranking Systems (part 2)

HS Rankingaufgaben in der Computerlinguistik

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UNIVERSITÄT ZU KÖLN

Recap

Learn to rank

- Features represent a pair of query and offering $\vec{x} = \phi(q_i, o_j)$
- Different ways of casting the problem
  - Pointwise: Model predicts y for individual $\vec{x}$, with y being an ordinal class or score
    - E.g., the model predicts that $\phi(q_4, o_1)$ get a score of 3
  - Pairwise: Model predicts, which of $\vec{x}_a$ and $\vec{x}_b$ are ranked higher
    - E.g., the model predicts that $\phi(q_9, o_3)$ comes before $\phi(q_9, o_7)$
  - Listwise: Model predicts the full ranking over a list of feature vectors $\vec{x}$
    - E.g., model predicts the ranking $\langle \phi(q_3, o_1), \phi(q_3, o_9), \phi(q_7, o_1) \rangle$
- Point- and pairwise approaches can be implemented with a "standard" ML algorithm

Section 1

Linear/Logistic Regression

Regression

- Regression
  - Prediction of numeric values (e.g., future COVID-19 cases; number of nouns in a text, ...)
  - Based on some input features (e.g., "R-Wert", number of past cases, ...)

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- Linear
  - The relation between input features and output values is linear
  - Math: $y = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + b$

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- Linear
  - The relation between input features and output values is linear
  - Math: $y = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + b$
- Logistic
  - Relation between input and output follows a logistic equation σ:
    - $0 \leq \sigma(x) \leq 1$, for all values of x
    - They can be interpreted as probabilities

Linear Regression

Example

- Input
  - Number of words in a (narrative, prose) text
- Output
  - Number of literary characters in the text
    (in the sense of “Figur”, not in the sense of “Zeichen”)

Linear Regression

Example

- Input
  - Number of words in a (narrative, prose) text
- Output
  - Number of literary characters in the text
    (in the sense of “Figur”, not in the sense of “Zeichen”)
- Linear equation (with one input variable): $y = ax + b$
  - With x being the number of tokens and y the number of characters
  - Real examples have more variables, but are harder to visualize

Linear Regression

Example scenario

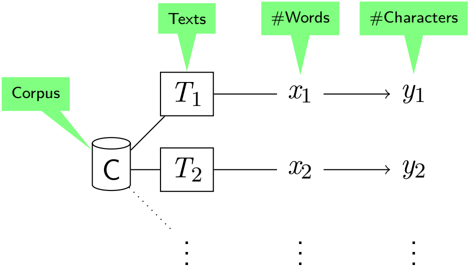

Figure: Schema of the example scenario

Linear Regression

The data set

x	y (# characters)
10	3
105	5
150	8
210	12
250	7
295	13

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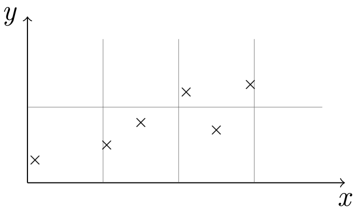

Figure: Data set, each $\times$ represents a text (x: text length, y: num. of characters)

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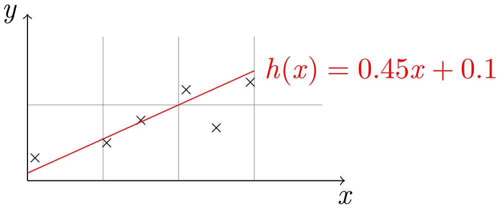

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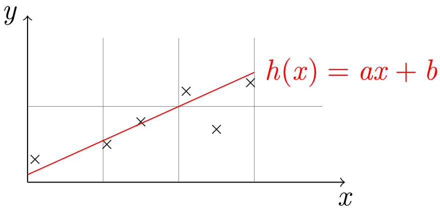
Linear Regression

The Task



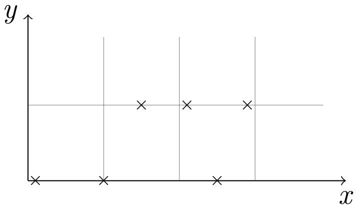
The Model

- Linear regression with one variable (= univariate linear regression)
- Prediction (hypothesis function): $y = h_{a,b}(x) = ax + b$
- How to set parameters a and b? $\rightarrow$ training algorithm

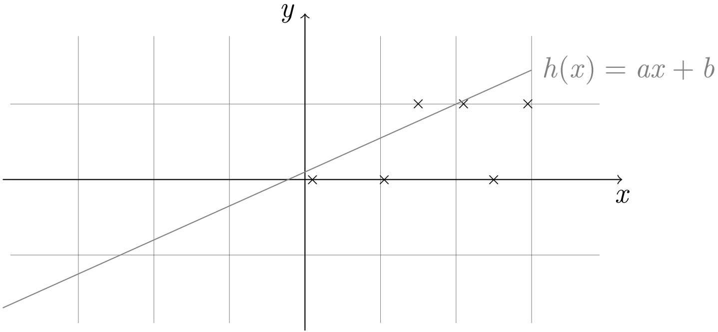
Doing Classification with Linear Regression

- Example task: Will a book receive a Nobel prize, given the number of literary characters in it?

# Characters	Win
1	No
10	No
15	Yes
21	Yes
25	No
29	Yes

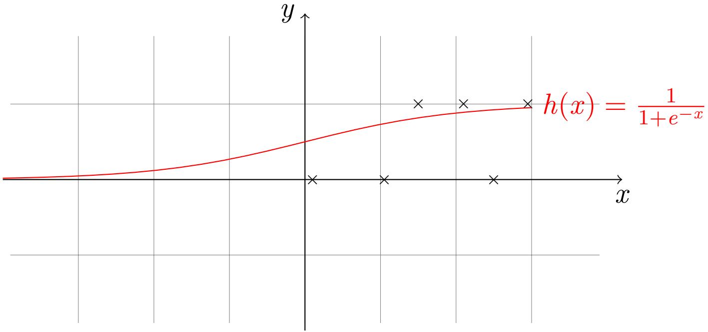


Fitting an Equation



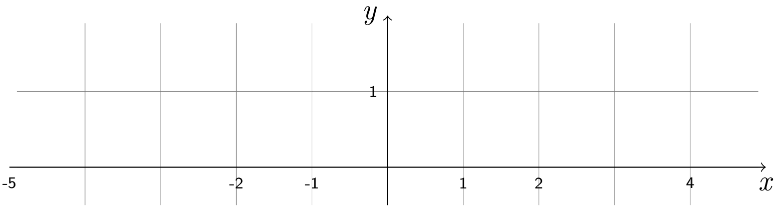
$\boxed{\text{linear equation}}$

Fitting an Equation



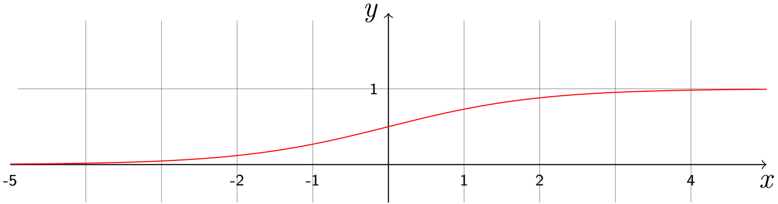
logistic function

The Logistic Function



$$y = \frac{1}{1+e^{-(ax+b)}} \qquad \text{(general form)}$$

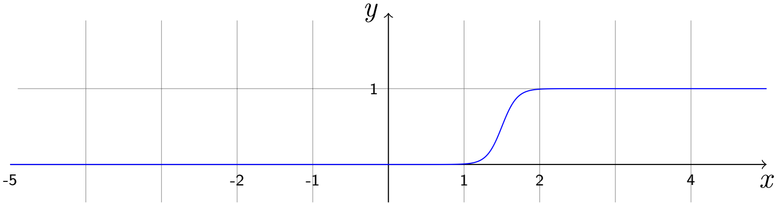
The Logistic Function



$y = \frac{1}{1+e^{-(ax+b)}}$ (general form)

$y = \frac{1}{1+e^{-(1*x+0)}}$

The Logistic Function



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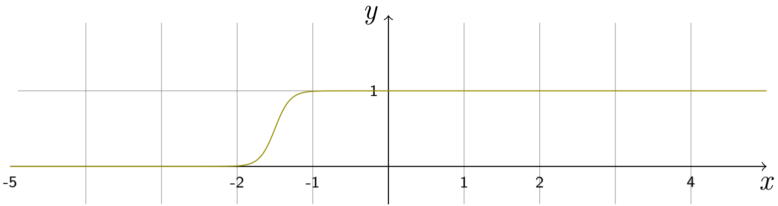

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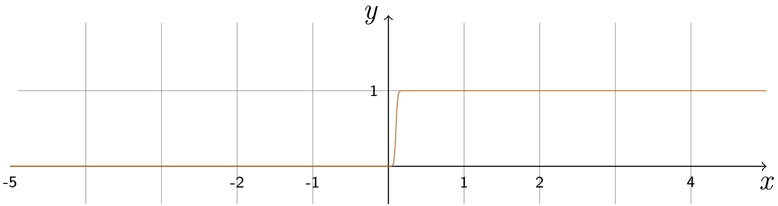

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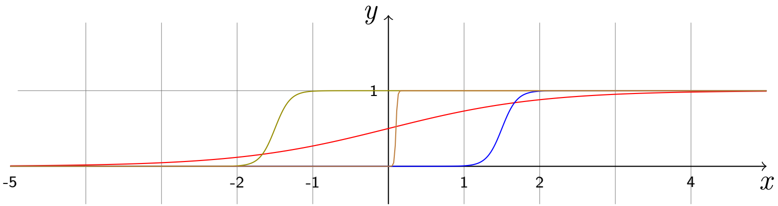

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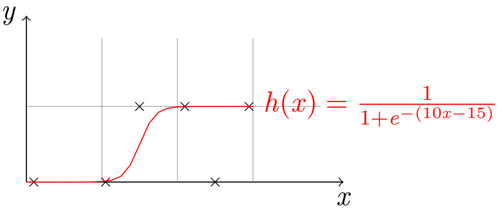
Parameter Fitting



- Linear equations can be wrapped in a logistic one
- Same parameters to be tuned (a and b)
- $e = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.71828$ (Euler's number)

Summary: Logistic Regression (with a single variable)

Logistic regression is half of the math of deep learning

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Logistic regression is half of the math of deep learning

- Regression: Predicting probabilities ➔ Binary classification
- Model
  - Logistic equations
  - $y = \frac{1}{1+e^{-(ax+b)}}$
- Learning algorithm: How to choose a and b?

Gradient Descent

Learning Regression Models

- How to select the parameters a, b such that the hypothesis function describes the data points as best as possible?
- Learning algorithm *Gradient Descent*

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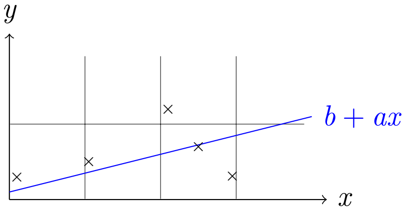
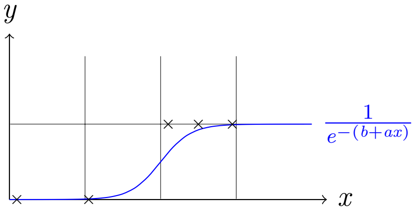
Gradient descent is half of the algorithms of deep learning

Loss: Intuition

The *loss* measures the 'wrongness' of values for a and b.





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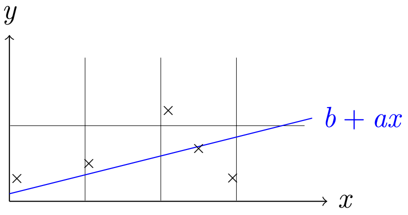

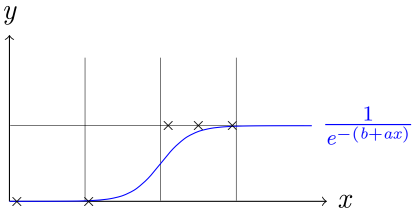

- How big is the gap between a hypothesis and the data?
- Is $(a, b) = (0.3, 0.5)$ or $(a, b) = (0.4, 0.4)$ better?

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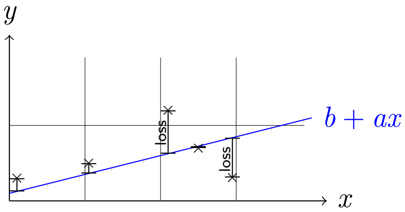

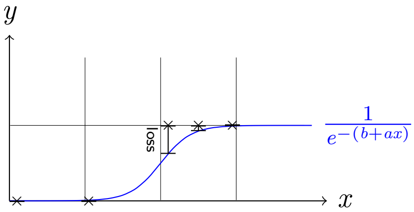

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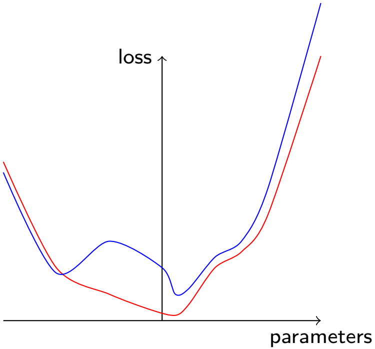
Loss function: Intuition

- Loss should be as small as possible
- Total loss can be calculated for given parameters $\vec{w} = (a, b)$
- Idea:
  - We change $\vec{w}$ until we find the minimum of the function
  - We use the derivative to find out if we are in a minimum
  - The derivative also tells us how to change the update parameters a and b

Loss function: Intuition



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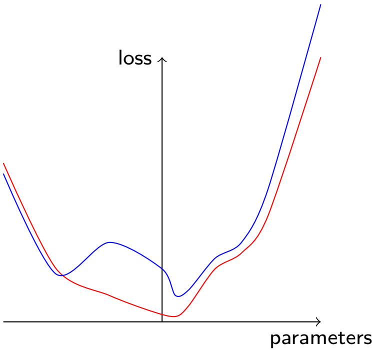

Function should be convex!
If not, we might get stuck in local minimum

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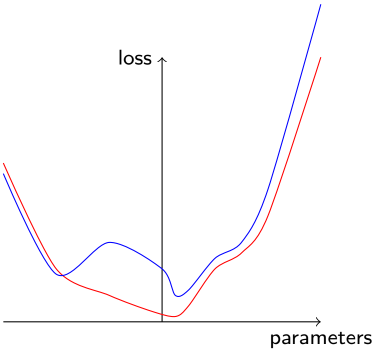

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Hypothesis vs. Loss Function

- Hypothesis function h
  - Calculates outcomes, given feature values x – and parameter values $\vec{w} = (a, b)$
- Loss function J
  - Calculates 'wrongness' of h, given parameter values $\vec{w}$ (and a data set)
  - In reality, $\vec{w}$ represents many more parameters (thousands)

Loss Function

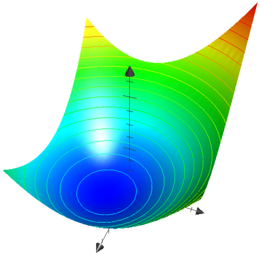

Figure: The loss function in our setting visualised

Loss Function

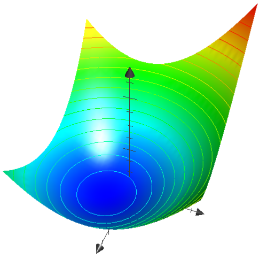

Figure: The loss function in our setting visualised

- Searching for the a, b settings with minimal loss
- $=$ Searching for the minimum!

Loss Function

Definition

Loss function depends on hypothesis function

Linear hypothesis function

- $h(x) = ax + b$
- Loss: Mean squared error

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Definition

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Linear hypothesis function

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Logistic hypothesis function

- $h(x) = \frac{1}{e^{-(b+ax)}}$
- Loss: (Binary) cross-entropy loss

Loss Function

Definition for Linear Regression

- The loss function is a function on parameter values a and b
  (for a given hypothesis function and data set)
  - Hypothesis function: $h_{\vec{w}} = w_1 x + w_0$

$\vec{w} = (a, b)$: parameters $h_{\vec{w}}$: hypothesis function m: number of items

$$J(\vec{w}) =$$

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$$\vec{w} = (a, b)\text{: parameters} \quad h_{\vec{w}}\text{: hypothesis function} \quad m\text{: number of items}$$

$$J(\vec{w}) = \qquad\qquad h_{\vec{w}}(x_i) - y_i$$

- Calculate the loss for item i

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$$J(\vec{w}) = \qquad (h_{\vec{w}}(x_i) - y_i)^2$$

- Calculate the loss for item i
- Square the error

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$$J(\vec{w}) = \sum_{i=1}^{m} (h_{\vec{w}}(x_i) - y_i)^2$$

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- Sum them up

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$$J(\vec{w}) = \frac{1}{m} \sum_{i=1}^{m} (h_{\vec{w}}(x_i) - y_i)^2$$

- Calculate the loss for item i
- Square the error
- Sum them up
- Divide by the number of items
  - Known as: *Mean squared error*

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  - Hypothesis function: $h_{\vec{w}} = w_1 x + w_0$

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$$J(\vec{w}) = \frac{1}{2}\frac{1}{m}\sum_{i=1}^{m}(h_{\vec{w}}(x_i) - y_i)^2$$

- Calculate the loss for item i
- Square the error
- Sum them up
- Divide by the number of items
  - Known as: *Mean squared error*
- Divide by two
  - out of convenience, because derivation

Loss function

Definition for Logistic Regression

- Two cases: $y_i = 0$ or $y_i = 1$ – y_i: real outcome for instance i
  - Caveat: $\log 0$ is undefined – add $\epsilon = 0.0000001$ if needed

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y_i	$h_{\vec{w}}(x_i)+\epsilon$	$y_i \log h_{\vec{w}}(x_i) + (1-y_i)\log(1-h_{\vec{w}}(x_i))$
0	1.0000001	−23.2535
0	0	0
1	1	0
1	0.0000001	−23.2535
1	0.8	−0.3219281
1	0.2	−2.321928

More Dimensions

- Above: 1 dimension, 2 parameters
  - a: slope, b: y-intercept
  - Input feature x, a single value

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  - Input vector $\vec{x}$ with $n - 1$ dimensions
  - Hypothesis function: $h_{\vec{w}}(x) = w_n x_n + w_{n-1} x_{n-1} + \ldots w_1 x_1 + w_0$
    - w_0: y-intercept, w_1 to w_n: slopes

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    - w_0: y-intercept, w_1 to w_n: slopes
- Algorithms
  - Derivatives more complicated
  - Otherwise identical

Section 2

Summary

Summary

Regression

- Fitting parameters to a data distribution
  - Linear R: Numeric prediction algorithm
    - Prediction model: $h_{\vec{w}}(x) = ax + b$
  - Logistic R: Classification algorithm (because we interpret results as probabilities)
    - Prediction model: $h_{\vec{w}}(x) = \frac{1}{e^{-(b+ax)}}$
- Learning algorithm: Gradient descent

Gradient Descent

- Initialise $\vec{w}$ with random values (e.g., 0)
- Repeat:
  - Find the direction to the minimum by taking the derivative
  - Change $\vec{w}$ accordingly, using a learning rate η
  - Stop when $\vec{w}$ don't change anymore