Deep Learning

Übung WS 23/24

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Recap

- Classification
  - Assign previously known labels to instances
  - Different algorithms, wide range of applications
- Logistic Regression
  - Classification method based on $h(x) = \frac{1}{1+e^{-(ax+b)}}$
  - Core ingredient of artificial neural networks
- Gradient Descent
  - Learning algorithm for logistic regression
  - Core ingredient of learning artificial neural networks
- `scikit-learn`
  - Python library for machine learning

Today

Neural Networks

Deep Learning Library `keras`

Exercise

Section 1

Neural Networks

From a Logistic Regression to a Neuron

- Hypothesis function of logistic regression:

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$\Rightarrow$ We can insert other functions as well!

What is a neural network?

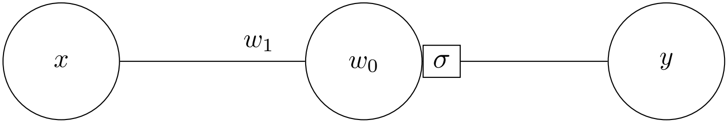

Figure: 1 neuron (with logistic activation) = logistic regression (with 1 feature)

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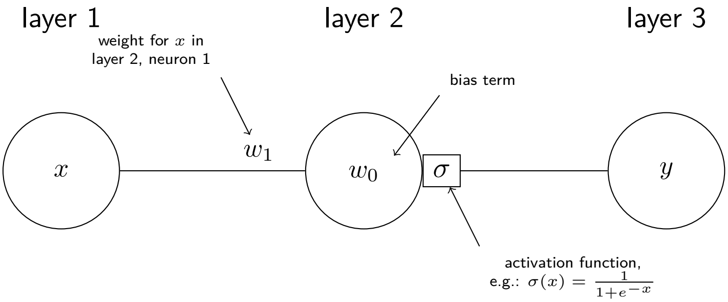

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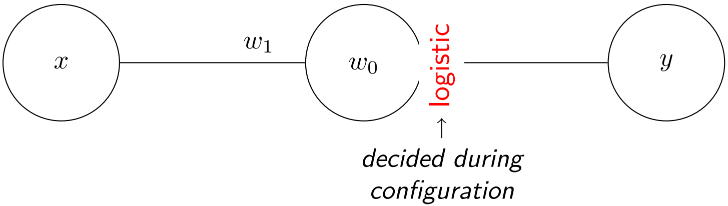

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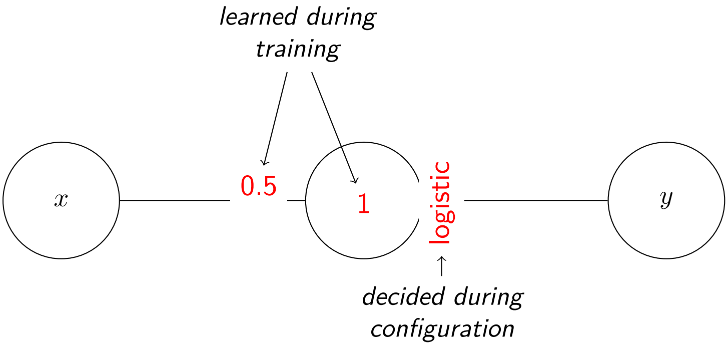

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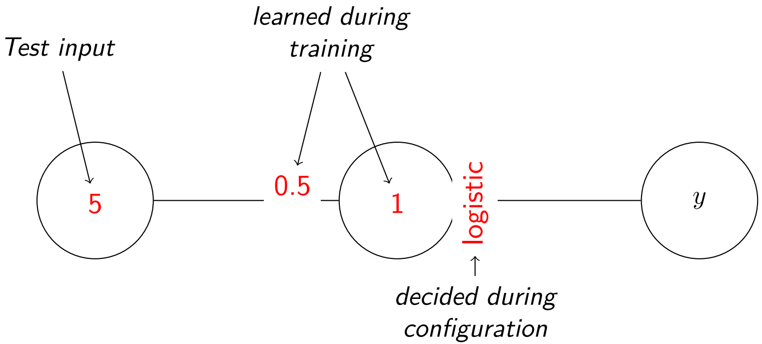

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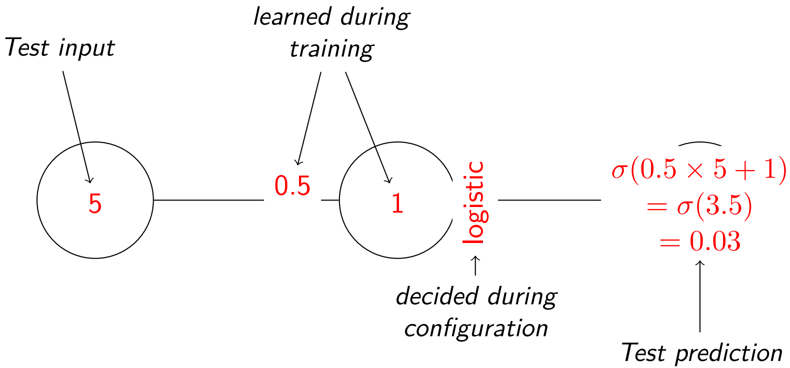

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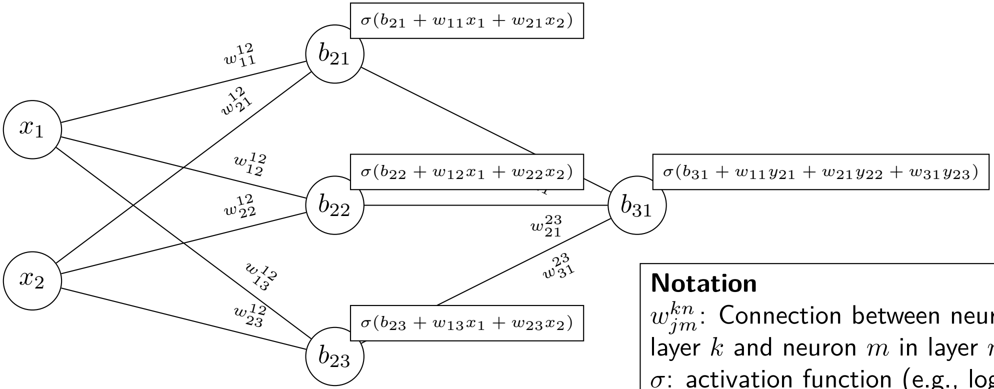

Notation
w_{jm}^{kn}: Connection between neuron j in layer k and neuron m in layer n
σ: activation function (e.g., logistic)

Figure: A simple neural network with 1 hidden layer

Prediction Model: Forward Pass

- If we have all the weights, bias terms, numbers of neurons and layers, we can compute the output of the network
  - Conceptually: Applying functions in sequence: $y = f_3(f_2(f_1(x)))$ (one per layer)

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- Practically, a lot of the computation happens in matrices
  - Hidden layer
    - Weights from input to hidden: $W_{1,2} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix}$
    - Biases $B_2 = (b_{21}, b_{22}, b_{23})$

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- Hidden layer computation
  - $f_2(X) = \sigma((W_{1,2}^{\intercal} X) + B_2)$
- Deep learning involves a lot of matrix multiplication
  - GPUs are highly optimized for this
  - Hint: Gaming-GPUs that support CUDA are also usable for deep learning

Feed-Forward Neural Networks

» The above is called a »feedforward neural network«
  - Information is fed only in forward direction

Feed-Forward Neural Networks

- The above is called a »feedforward neural network«
  - Information is fed only in forward direction
- Configuration choices
  - Activation function (next slide)
  - Layer size: Number of neurons in each layer
  - Number of layers
  - Loss function
  - Optimizer
- Training choices
  - Epochs/batches
  - Training status displays

Feed-Forward Neural Networks

Activation Functions

All neurons of one layer have the same
Popular choices:

- **logistic** $y = \sigma(x) = \frac{1}{1+e^{-x}}$ – ›squashes‹ everything to a value between 0 and 1
- **relu** $y = \max(0, x)$ – Makes everything negative to 0
- **softmax** Scales a vector such that values sum to 1 (probability distribution)

Training: »Backpropagation«

- Similar to gradient descent
- But
  - A lot more parameters
  - Because of multiple layers: Vanishing gradients
    - Backpropagation involves a lot of multiplication
    - Factors are between zero and one

    $\Rightarrow$ Numbers get very small very quickly

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- Training choice: Batches and epochs

Training a Feedforward Neural Network I

Stochastic Gradient Descent (SGD)

- Gradient Descent
  - Apply θ to all training instances
  - Calculate loss over entire data set
- Stochastic Gradient Descent
  - Data set in random order
  - Calculate loss for every single instance, then update weights

Training a Feedforward Neural Network II

When to stop the training

- Logistic regression (last week): Stop in minimum
- In theory, that's what we want
- In practice
  - We usually are not exactly in the minimum
  - It's not important to be exactly in the minimum

$\Rightarrow$ Fixed number of iterations over the data set (= number of epochs)

Batches vs. Epochs

batch Number of instances used before updating weights

epochs Number of iterations over all instances

Section 2

Deep Learning Library `keras`

keras

- https://keras.io
- High-level Python API for deep learning
- Built on top of tensorflow
- Pattern: 1. Layout the network, 2. set hyper parameters, 3. run training

Listing: Installing Keras

```
pip install keras
```

Configuration

- » Sequential API: Linear topology of layers
  - We will use mainly this
- » Functional API: Graph of layers

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Listing: Sequential API

```
# model layout
model = Sequential()
model.add(...)
model.add(...)

# hyperparameter specification
model.compile(loss=...,
optimizer=...)

# training
model.fit(..., epochs=...,
batch_size=...)
```

Listing: Functional API

```
# model layout
in = ...
out = Dense(10)(in)
model = Model(inputs=in,
outputs=out)

# hyperparameter specification
model.compile(loss=...,
optimizer=...)

# training
model.fit(..., epochs=...,
batch_size=...)
```

Configuration

Two most basic layer types

- Dense: »Just your regular densely-connected NN layer.«
  - https://keras.io/api/layers/core_layers/dense/

```
layer = Dense(3, # number of neurons
activation = activations.sigmoid, # activation function
name = "dense layer 7" # useful for debugging/visualisation
... # more options, see docs
)
```

- Input: Marks layers to accept data
  - https://keras.io/api/layers/core_layers/input/

```
layer = Input(shape=(15,) # number of input dimensions/features
name = "input layer", # useful for debugging/visualisation
... # see docs
)
```

Shape

- Description of the dimensionality of the data
- A vector of numbers, giving the number of elements for each dimension
- Python tuple
  - List with fixed length: `x = (5,3,1) #a tuple`
  - ⚠ Tuple with one element printed as `(5,)` or `5`

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```
x = np.zeros(5) # array([0., 0., 0., 0., 0.])
x.shape # returns (5,)
x = np.zeros((3,5))
# array([[0., 0., 0., 0., 0.],
#        [0., 0., 0., 0., 0.],
#        [0., 0., 0., 0., 0.]])
x.shape # returns (3,5)
```

A Full Example

```
import numpy as np
from tensorflow import keras
from tensorflow.keras import layers

# create a random data set
train = np.random.randn(75)
train = train.reshape([3, 25])
y_train = train[0, :]
x_train = np.rot90(train[0, :])

# setup the model architecture
model = keras.Sequential()
model.add(layers.Input(shape=(3,)))
model.add(layers.Dense(5, activation="sigmoid"))
model.add(layers.Dense(1, activation="softmax"))

# compile it
model.compile(loss="mean_squared_error",optimizer="sgd",metrics=["accuracy"])

# train it
model.fit(x_train, y_train, epochs=100, batch_size=5)
```

Section 3

Exercise

Exercise 06

https://github.com/IDH-Cologne-Deep-Learning-Uebung/exercise-06