UNIVERSITÄT
ZU KÖLN

DECISION TREES

Sprachverarbeitung (Vorlesung)

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15 May 2025

*Based on slides by Nils Reiter

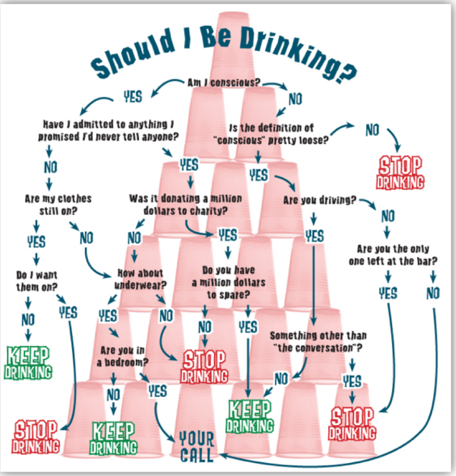
Recap

- Evaluation of machine learning models
- Accuracy, error rate
  - Single score for entire classification
- Precision, Recall, F-Score
  - Scores for each class
  - Precision: How many of the items classified as c are truly category c?
  - Recall: How many of the items that are truly c did the system find?
- Baseline

01

DECISION TREES

Prediction Model – Toy Example



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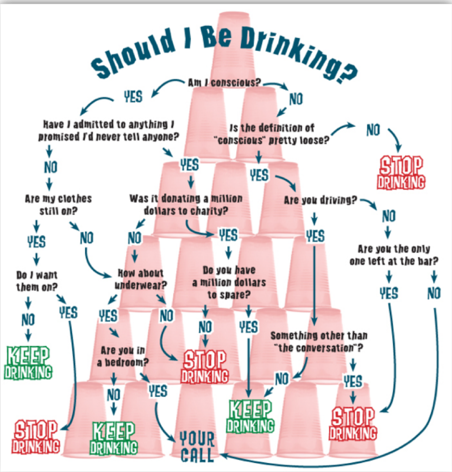

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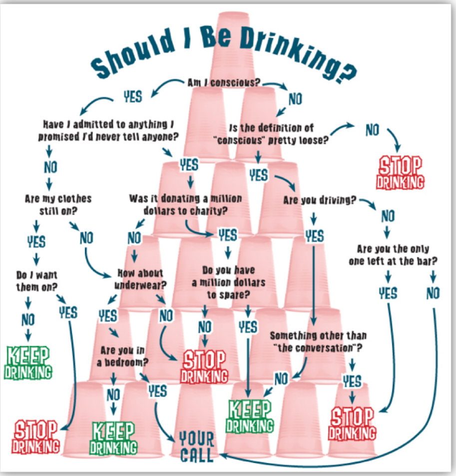

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  - Situations we are in
    (this is not really automatisable)

Prediction Model – Toy Example

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- What are the features?

Prediction Model – Toy Example

- What are the instances?
  - Situations we are in
    (this is not really automatisable)
- What are the features?
  - Consciousness
  - Clothing situation
  - Promises made
  - Whether we are driving
  - …

Trees

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  - some value and
  - a (possibly empty) set of children
    - Children are also trees

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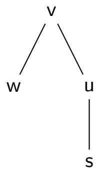

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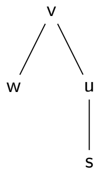
- Formally: $\langle v, \{\langle w, \emptyset\rangle, \langle u, \{\langle s, \emptyset\rangle\}\rangle\}\rangle$
- Recursive definition: "A tree is something and a bunch of sub trees"
  - Recursion is an important ingredient in many algorithms and data structures



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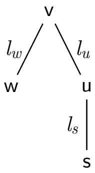
- Recursive definition: “A tree is something and a bunch of sub trees”
  - Recursion is an important ingredient in many algorithms and data structures
- If the tree has labels on the edges, the pair becomes a triple
  - $\langle v, \emptyset, \{\langle w, l_w, \emptyset\rangle, \langle u, l_u, \{\langle s, l_s, \emptyset\rangle\}\rangle\}\rangle$



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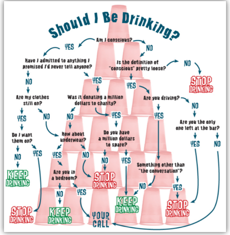
  - Number of branches $= |v(f_i)|$ (number of possible values)
- Each leaf node represents a class label
- Make a prediction for x:
  1. Start at root node
  2. If it's a leaf node
     - assign the class label
  3. Else
     - Check node which feature is to be tested (f_i)
     - Extract $f_i(x)$
     - Follow corresponding branch
     - Go to 2



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     - Done.
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     - Select a feature f_i
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- Remaining question: How to select features?

Feature Selection

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    - One that maximizes homogeneity in the split data set

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- Homogeneity: Entropy/information (Shannon 1948)
- Rule: Always select the feature with the highest *information gain* (IG)
  - (= the highest reduction in entropy = the highest increase in homogeneity)

Entropy

Intuition

- Measures the amount of uncertainty
- How uncertain is the next symbol in these sequences?
  - aaaaaaaaaaaaaa

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  - cbabcababcbaca – three symbols, evenly distributed, 33:66
  - nmkfjigeahldcb – 14 symbols, very uncertain
- Certainty depends on number of different symbols and on their distribution

Entropy (Shannon 1948)

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i)$$

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$\log_b(x) = y$
exactly if
$b^y = x$:
$2^5 = 32 \Leftrightarrow \log_2 32 = 5$

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Interpretation

Entropy is the average number of bits* we need to specify an outcome of the random variable
(* for $b = 2$)

Entropy (Shannon 1948)

Examples

$$
\begin{aligned}
H(X) &= -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i) \\
H(\{\spadesuit\spadesuit\spadesuit\spadesuit\}) &= -\frac{4}{4}\log_2\frac{4}{4} = 0 \\
H(\{\spadesuit\spadesuit\spadesuit\heartsuit\}) &= -\left(\underbrace{\frac{3}{4}\log_2\frac{3}{4}}_{\spadesuit} + \underbrace{\frac{1}{4}\log_2\frac{1}{4}}_{\heartsuit}\right) = 0.811 \\
H(\{\spadesuit\spadesuit\heartsuit\heartsuit\}) &= \ldots = 1 = H(\{\spadesuit\spadesuit\spadesuit\heartsuit\heartsuit\heartsuit\}) = \ldots
\end{aligned}
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H(\{\spadesuit\spadesuit\heartsuit\heartsuit\clubsuit\clubsuit\}) &= 1.585 \\
H(\{\spadesuit\heartsuit\clubsuit\diamondsuit\}) &= 2 \\
H(\{nmkfjigeahldcb\}) &= 3.807
\end{aligned}
$$

Entropy

Mutual Information

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  - Joint entropy: Amount of uncertainty in two random variables
  - Conditional entropy: Amount of uncertainty, when another random variable is known

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- Mutual Information (Information Gain)
  - Reduction of entropy in one random variable by knowing about the other
  - $MI(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}$

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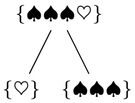
- Point-wise Mutual Information
  - Statement about values of random variable (i.e., occurrence of specific word)
  - $PMI(w_1, w_2) = \log_2 \frac{p(w_1, w_2)}{p(w_1)p(w_2)}$

MS99, p. 67

Feature Selection



$$
\begin{aligned}
H(\{\spadesuit\spadesuit\spadesuit\heartsuit\}) &= H([3,1]) = 0.562 \\
H(\{\heartsuit\}) &= H([1]) = 0 \\
H(\{\spadesuit\spadesuit\spadesuit\}) &= H([3]) = 0
\end{aligned}
$$

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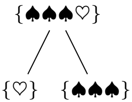

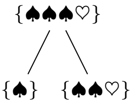

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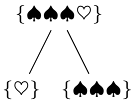

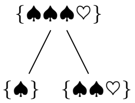

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$$\begin{aligned} IG(f_1) &= H(\{\spadesuit\spadesuit\spadesuit\heartsuit\}) - \varnothing\big(H(\{\heartsuit\}), H(\{\spadesuit\spadesuit\spadesuit\})\big) \\ &= 0.562 - 0 = 0.562 \\ IG(f_2) &= H(\{\spadesuit\spadesuit\spadesuit\heartsuit\}) - \varnothing\big(H(\{\spadesuit\}), H(\{\spadesuit\spadesuit\heartsuit\})\big) \\ &= 0.562 - (\frac{3}{4}0.637 + \frac{1}{4}0) \\ &= 0.562 - 0.477 = 0.085 \end{aligned}$$

Feature Selection using Entropy

- We calculate entropy for the target class
- But in different sub sets of the data set

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Code Listing 2: Feature selection in pseudo code for a data set D

```
function select_feature(D):
  base_entropy = entropy(D)
  ig_map = {}
  foreach feature f:
    weighted_feature_entropy = 0
    foreach feature value v:
      D_v = subset of D with all instances that have the value v
      sub_entropy = entropy(D_v)
      sub_size = length(D_v)
      weighted_feature_entropy = weighted_feature_entropy + ( sub_entropy * sub_size )
    information_gain = base_entropy − ( (weighted_feature_entropy) / length(D) )
    ig_map.put(f, information_gain)
  return maximum from ig_map
```

ID3

J. Ross Quinlan (Mar. 1986). "Induction of Decision Trees". In: *Machine Learning* 1.1, pp. 81–106. DOI: 10.1007/BF00116251

Limitations

- Only categorical attributes
- Cannot handle missing values
- Tends to overfit: "In my experience, almost all decision trees can benefit from simplification" (Quinlan 1993, p. 36)
  - Even today, overfitting is a huge challenge for ML algorithms!

⇒ Extension: C4.5 (Quinlan 1993)

Data set

- Data set: 100 e-mails, manually classified as spam or not spam (50/50)
  - Classes $C = \{\text{true}/1, \text{false}/0\}$
- Features: Presence of each of these tokens (manually selected): 'casino', 'enlargement', 'meeting', 'profit', 'super', 'text', 'xxx'

Mail	'casino'	'enlargement'	'meeting'	'profit'	'super'	'text'	'xxx'	C
1	1	1	0	0	1	1	1	0
2	0	1	0	1	0	0	0	1
3	1	0	1	0	1	0	0	0
4	1	1	1	0	0	0	0	0
5	0	1	1	0	0	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Learning Algorithm

First step: Use the full data set

$$H(\text{full data set}) = 1$$

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$$\begin{aligned} H(\text{full data set}) &= 1 \\ H(\text{`casino'} = 1) &= 0.9991 \\ H(\text{`casino'} = 0) &= 0.9985 \end{aligned}$$

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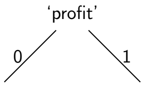
H(\text{full data set}) &= 1 \\
H(\text{`casino'} = 1) &= 0.9991 \\
H(\text{`casino'} = 0) &= 0.9985 \\
H(\text{`casino'}) &= \frac{(56 \times 0.9991) + (44 \times 0.9985)}{100} = 0.9989 \\
IG(\text{`casino'}) &= 1 - 0.9989 = 0.0012 \\
IG(\text{`profit'}) &= 0.0073 \\
&\vdots \qquad \vdots
\end{aligned}
$$

Learning Algorithm



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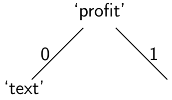
Learning Algorithm

Next step: Use the data set *after* application of the first selected feature
'profit' = 0

$$
\begin{aligned}
H(\text{data set}) &= 0.99403 \\
H(\text{`casino'} = 1) &= 0.9910 \\
H(\text{`casino'} = 0) &= 0.9963 \\
IG(\text{`casino'}) &= 0.00029 \\
IG(\text{`text'}) &= 0.01151
\end{aligned}
$$

Learning Algorithm



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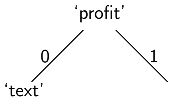
$$\begin{aligned} H(\text{data set}) &= 0.99107 \\ H(\text{‘casino’} = 1) &= 0.9366 \\ H(\text{‘casino’} = 0) &= 1 \\ IG(\text{‘casino’}) &= 0.0150 \\ IG(\text{‘meeting’}) &= 0.00029 \end{aligned}$$

Learning Algorithm



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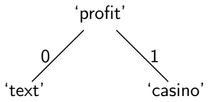
$$\begin{aligned} H(\text{data set}) &= 0.99403 \\ H(\text{'casino'} = 1) &= 0.9910 \\ H(\text{'casino'} = 0) &= 0.9963 \\ IG(\text{'casino'}) &= 0.00029 \\ IG(\text{'text'}) &= 0.01151 \end{aligned}$$

'profit' = 1

$$\begin{aligned} H(\text{data set}) &= 0.99107 \\ H(\text{'casino'} = 1) &= 0.9366 \\ H(\text{'casino'} = 0) &= 1 \\ IG(\text{'casino'}) &= 0.0150 \\ IG(\text{'meeting'}) &= 0.00029 \end{aligned}$$

Learning Algorithm



Next step: Use the data set *after* application of the first selected feature

'profit' = 0

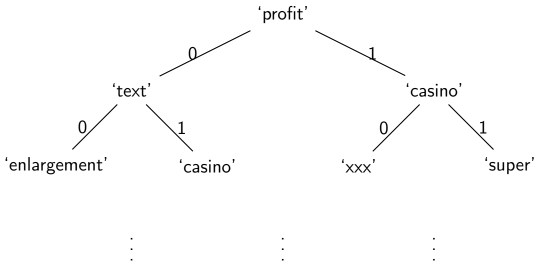
$$\begin{aligned} H(\text{data set}) &= 0.99403 \\ H(\text{`casino'} = 1) &= 0.9910 \\ H(\text{`casino'} = 0) &= 0.9963 \\ IG(\text{`casino'}) &= 0.00029 \\ IG(\text{`text'}) &= 0.01151 \end{aligned}$$

'profit' = 1

$$\begin{aligned} H(\text{data set}) &= 0.99107 \\ H(\text{`casino'} = 1) &= 0.9366 \\ H(\text{`casino'} = 0) &= 1 \\ IG(\text{`casino'}) &= 0.0150 \\ IG(\text{`meeting'}) &= 0.00029 \end{aligned}$$

Learning Algorithm

Next step: Use the data set *after* application of the first two layers of selected features



02

SUMMARY

Summary

- Decision Tree
  - Transparent prediction model: Easy to apply by humans
  - Easy to implement: Follow the path form root to leaf
  - Learning algorithm
    - Recursively split the training data set according to features
    - Use information gain to maximize the homogeneity in the sub sets

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